Factor Analysis, Path Analysis, and Structural Equation Modeling

CHAPTER

14

■ Introduction

Up to this point in the discussion of multivariate statistics, we have focused on the relationship of isolated independent variables on a single dependent variable at some level of measurement. In criminological theory, theorists are often concerned a collection of variables, not with observable individual variables. In these cases, regression analysis alone is often inadequate or in appropriate. As such, when testing theoretical models and constructs, as well as when examining the interplay between independent variables, other multivariate statistical analyses should be used.

This chapter explores three multivariate statistical techniques that are more effective and more appropriate for analyzing complex theoretical models. The statistical applications examined here are factor analysis, path analysis, and structural equation modeling.

Factor Analysis

Factor analysis is a multivariate analysis procedure that attempts to identify any underlying "factors" that are responsible for the covariaton among a group independent variables. The goals of a factor analysis are typically to reduce the number of variables used to explain a relationship or to determine which variables show a relationship. Like a regression model, a factor is a linear combination of a group of variables (items) combined to represent a scale measure of a concept. To successfully use a factor analysis, though, the variables must represent indicators of some common underlying dimension or concept such that they can be grouped together theoretically as well as mathematically. For example, the variables income, dollars in savings, and home value might be grouped together to represent the concept of the economic status of research subjects.

Factor analysis originated in psychological theory. Based on the work undertaken by Pearson (1901) in which he proposed a ". . . method of principal axes . . .", Spearman (1904) began research on the general and specific factors of intelligence. Spearman's two factor model was enhanced in 1919 with the development by Garnett of a multiple-factor approach. This multiple-factor model was officially coined "factor analysis" by Thurstone in 1931.

There are two types of factor analyses: **exploratory** and **confirmatory**. The difference between these is much like the difference discussed in regression between testing a model without changing it and attempting to build the best model based on the data utilized.

Exploratory factor analysis is just that: exploring the loadings of variables to try to achieve the best model. This usually entails putting variables in a model where it is expected they will group together and then seeing how the factor analysis groups them. At the lowest level of science, this is also commonly referred to as "hopper analysis," where a large number of variables are dumped into the hopper (computer) to see what might fit together; then a theory is built around what is found.

At the other end of the spectrum is the more rigorous confirmatory factor analysis. This is confirming previously defined hypotheses concerning the relationships between variables. In reality, probably the most common research is conducted using a combination of these two where the researcher has an idea which variables are going to load and how, uses a factor analysis to support these hypotheses, but will accept some minor modifications in terms of the grouping.

It is common for factor analysis in general, and exploratory factor analysis specifically, to be considered a **data reduction** procedure. This entails placing a number of variables in a model and determining which variables can be removed from the model; making it more parsimonious. Factor analysis purists decry this procedure, holding that factor analysis should only be confirmatory; confirming what has previously been hypothesized in theory. Any reduction in the data/variables at this point would signal weakness in the theoretical model.

There are other practical uses of factor analysis beyond what has been discussed above. First, when using several variables to represent a single concept (theoretically), factor analysis can confirm that concept by its identification as a factor. Factor analysis can also be used to check for multicollinearity in variables to be used in a regression analysis. Variables which group together and have high factor loadings are typically multicollinear. This is not a common method of determining multicollinearity as it adds another layer of analysis.

There are two key concepts to factor analysis as a multivariate analysis technique: **variance** and **factoral complexity**. Variance is discussed in the four paragraphs below, followed by a discussion of factoral complexity.

Variance comes into play because factor analysis attempts to identify factors that explain as much of the common variance within a set of variables as possible. There are three components to variance: **communality**, **uniqueness** and **error variance**.

Communality is the part of the variance shared with one or more other variables. Communality is represented by the sum of the squared loadings for a variable (across factors). Factor analysis attempts to determine the factor or factors that explain as much of the communality of a set of variables as possible. All of the variance in a set of variables can be explained if there are as many factors as variables. That is not the goal of factor analysis, however. Factor analysis attempts to explain as much of the variance as possible with the least amount of variables (parsimony). This will become important in the interpretation of factor analysis.

Uniqueness, on the other hand, is the variance specific to a particular variable. Part of the variance in any model can be attributed to variance in each of the component variables (communality). Part of the variance, however, is unique to the specific factor

and cannot be explained by the component variables. Uniqueness measures the variance that is reflected in a single variable alone. It is assumed to be uncorrelated with the component factors or with other unique factors.

Error variance is the variance due to random or systematic error in the model. This is in line with the error that was discussed in Chapter 11, and the error associated with regression analysis discussed in the preceding chapters.

Factoral complexity is the number of variables loading on a given factor. Ideally a variable should only load on one factor. This means, theoretically, that you have accurately determined conceptually how the variables will group. The logical extension of this is that you have a relatively accurate measure of the underlying dimension (concept) that is the focus of the research. Variables that load (cross-load) on more than one factor represent a much more complex model, and it is more difficult to determine the true relationships between variables, factors, and the underlying dimensions.

■ Assumptions

As with the other statistical procedures discussed in this book, there are several assumptions that must be met before factor analysis can be utilized in research. Many of these are similar to assumptions discussed in previous chapters; but some are unique to factor analysis.

Like regression, the most basic assumption of factor analysis is that the data is interval level and normally distributed (linearity). Dichotomized data can be used with a factor analysis; but it is not widely accepted. It is acceptable to use dichotomized, nominal level data for the principle component analysis portion of the factor analysis. There is debate among statisticians, however, whether dichotomized, nominal level data is appropriate after the factors have been rotated because of the requirement of the variables being a true linear combination of the underlying factors. It has been argued, for example, that using dichotomized data yields two orthogonal (uncorrelated) factors such that the resulting factors are a statistical artifact and the model is misspecified. Other than dichotomized nominal level data, nominal and ordinal level data should not be used with a factor analysis. The essence of a factor analysis is that the factor scores are dependent upon the data varying across cases. If it cannot be reasoned that nominal level data varies (high to low or some other method of ordering) then the factors are uninterpretable. It is also not advised to use ordinal level data because of the non-normal nature of the data. The possible exception to this requirement concerns fully ordered ordinal level data. If the data can be justified as approximating interval level (as some will do with Likert type data), then it is appropriate to use it in a factor analysis.

The second assumption is that there should be no specification error in the model. As discussed in previous chapters, specification error refers to the exclusion of relevant variables from the analysis or the inclusion of irrelevant variables in the model. This is a severe problem for factor analysis that can only be rectified in the conceptual stages of research planning.

Another requirement of factor analysis is that there be a sufficient sample size so there is enough information upon which to base analyses. Although there are many different thoughts on how large the sample size should be for an adequate factor analysis, the general guidelines follow those of Hatcher (1994) who argued for sample sizes

of at least 100, or 5 times the number of variables to be included in the principle component analysis. Any of these kinds of cut-points are simply guides, however, and the actual sample size required is more of a theoretical and methodological issue for individual models. In fact, there are many who suggest that factor analysis is stable with sample sizes as small as 50.

One major difference in the assumptions between regression and factor analysis is multicollinearity. In regression, multicollinearity is problematic; in factor analysis, multicollinearity is necessary because variables must be highly associated with some of the other variables so they will load ("clump") into factors. The only caveat here is that all of the variables should not be highly correlated or only one factor will be present (the only criminological theory that proposed one factor is Gottfredson and Hirschi's work on self-control theory). It is best for factor analysis to have groups of variables highly associated with each other (which will result in those variables loading as a factor) and not correlated at all with other groups of variables.

Analysis and Interpretation

As with most multivariate analyses, factor analysis requires a number of steps to be followed in a fairly specific order. The steps in this process are outlined in **Table 14-1** and discussed below.

Step 1—Univariate Analysis

As with other multivariate analysis procedures, proper univariate analysis is important. It is particularly important to examine the nature of the data as discussed in Chapter 10. The examination of the univariate measures of skewness and kurtosis will be important in this analysis. If a distribution is skewed or kurtose, it may not be normally distributed and/or non-linear. This is detrimental to factor analysis. Any variables that are skewed or kurtose should be carefully examined before being used in a factor analysis.

Step 2—Preliminary Analysis

Beyond examining skewness and kurtosis, there are a number of preliminary analyses that can be used to ensure the data and variables are appropriate for a factor analysis. These expand upon the univariate analyses and are more specific to factor analysis.

TABLE 14-1   **Steps in the Factor Analysis Process**

Step 1	Examine univariate analysis of the variables to be included in the factor analysis
Step 2	Preliminary analyses and diagnostic tests
Step 3	Extract factors
Step 4	Factor extraction
Step 5	Factor rotation
Step 6	Use of factors in other analyses

How Do You Do That?
Obtaining Factor Analysis Output in SPSS

1. Open a data set such as one provided on the CD in the back of this book.
   a. Start SPSS
   b. Select *File*, then *Open*, then *Data*
   c. Select the file you want to open, then select *Open*
2. Once the data is visible, select *Analyze*, *Data Reduction*, *Factor . . .*
3. Select the independent variables you wish to include in your factor analysis your dependent variable and press the ► next to the *Variables* window.
4. Select the *Descriptives* button and click on the boxes next to *Anti-image* and *KMO and Bartlett's Test of Spericity* and select *Continue*.
5. Select the *Extraction* button and click on the box next to *Scree Plot* and select *Continue*.
6. Select *Rotation* and click on the box next to the type of *Rotation* you wish to use and select *Continue*.
7. Click *OK*.
8. An output window should appear containing tables similar in format to the tables below.

First, a **Bartlett's Test of Sphericity** can be used to determine if the correlation matrix in the factor analysis is an **Identity Matrix**. An identity matrix is a correlation matrix where the diagonals are all 1 and the off-diagonals are all 0. This would mean that none of the variables are correlated with each other. If the Bartlett's Test is not significant, do not use factor analysis to analyze the data because the variables will not load together properly. In the example in **Table 14-2**, the Bartlett's Test is significant, so the data meets this assumption.

It is also necessary to examine the **Anti-Image Correlation Matrix** (**Table 14-3**). This shows if there is a low degree of correlation between the variables when the other variables are held constant. Anti-image means that low correlation values will produce large numbers. The values to be examined for this analysis are the off diagonal values; the diagonal values will be important for the KMO analysis below. In the anti-image matrix in Table 14-3, the majority of the off-diagonal values are closer to zero. This is

TABLE 14-2   KMO and Bartlett's Test

Kaiser-Meyer-Olkin Measure of Sampling Adequacy.		.734
Bartleffs Test of Sphericity	Approx. Chi-Square	622.459
	df	91
	Sig.	.000

TABLE 14-3   Anti-image Correlation Matrix*

	ARR ESTR	CHU RCH	CLUBS	CUR FEW	GANG R	GRND DRG	GUN REG	OUT DRUG	OUT GUN	GRUP GUN	UCON VICR	U_GUN _CM	SKIP PEDR	SCH DRUG
ARRESTR	.545(a)													
CHURCH	-.010	.648(a)												
CLUBS	-.0412	-.166	.734(a)											
CURFEW	-.055	-.215	-.059	.695(a)										
GANGR	-.141	-.054	.029	.017	.849(a)									
GRND_DRG	.000	.233	-.066	-.160	-.104	.598(a)								
GUN_REG	.076	.041	-.022	.051	-.250	-.125	.768(a)							
OUT_DRUG	.082	.092	-.021	-.024	-.154	.063	-.338	.733(a)						
OUT_GUN	.032	-.004	-.133	.152	-.243	.076	-.036	-.009	.819(a)					
GRUP_GUN	-.015	-.026	.015	.154	-.161	-.121	.063	.052	-.229	.787(a)				
UCONVICR	-.286	.022	.181	.175	-.070	-.252	.128	.087	-.051	.052	.667(a)			
U_GUN_CM	.196	-.116	.052	-.082	-.119	.166	-.316	-.049	-.153	-.022	-.514	.682(a)		
SKIPPEDR	.079	-.209	-.125	-.014	.106	.035	.043	-.023	-.012	.106	.047	-.011	.842(a)	
SCH_DRUG	-.208	-.022	.163	-.042	.038	-.081	-.039	-.323	-.153	-.034	-.062	.085	.03	.727(a)

*See the Appendix C for full description of the variables used in Table 14–2.

what we want to see. If there are many large values in the off-diagonal, factor analysis should not be used.

Additionally, this correlation matrix can be used to asses the adequacy of variables for inclusion in the factor analysis. By definition, variables that are not associated with at least some of the other variables will not contribute to the analysis. Those variables identified as having low correlations with the other variables should be considered for elimination from the analysis (dependent, of course, on theoretical and methodological considerations).

In addition to determining if the data is appropriate for a factor analysis, you should determine if the sampling is adequate for analysis. This is accomplished by using the Kaiser-Meyer-Olkin Measure of Sampling Adequacy (Kaiser 1974a). The KMO compares the observed correlation coefficients to the partial correlation coefficients. Small values for the KMO indicate problems with sampling. A KMO value of 0.90 is best; below 0.50 is unacceptable. A KMO value that is less than .50, means you should look at the individual measures that are located on the diagonal in the anti-image matrix. Variables with small values should be considered for elimination. In the example in Table 14-2, the KMO value is .734. This is an acceptable KMO value, although it may be useful to examine the anti-image correlation matrix to see what variables might be bringing the KMO value down. For example, the two diagonal values that are between 0.50 and 0.60 are ARRESTR (Have you ever been arrested?) and GRND_DRG (Have you ever been grounded for drugs or alcohol?). ARRESTR cannot be deleted from the analysis because it is one of the dependent variables, but it might be necessary to determine whether GRND_DRG should be retained in the analysis.

Step 3—Extract the Factors

The next step in the factor analysis is to extract the factors. The most popular method of extracting factors is called a **principle component analysis** (developed by Hotelling, 1933). There are other competing, and sometimes preferable, extraction methods (such as **maximum likelihood**, developed by Lawley in 1940). These analyses determine how well the factors explain the variation. The goal here is to identify the linear combination of variables that account for the greatest amount of common variance.

As shown in the principal components analysis in **Table 14-4**, the first factor accounts for the greatest amount of common variance (26.151%), representing an Eigenvalue of 3.661. Each subsequent factor explains a portion of the remaining variance until a point is reached (an Eigenvalue of 1) where it can be said that the factors no longer contribute to the model. At this point, those factors with an Eigenvalue above 1 represent the number of factors needed to describe the underlying dimensions of the data. For Table 14-4, this is factor 5, with an explained variance of 7.658 and an Eigenvalue of 1.072. All of the factors below this do not continrue an adequate amount to the model to be included. Each of the factors at this point are not correlated with each other (they are orthogonal as described below).

Note here that a principal components analysis is **not** the same thing as a factor analysis (the rotation part of this procedure). They are similar, but the principal components analysis is much closer to a regression analysis, where the variables themselves are examined and their variance measured. Also note that this table lists numbers and not variable names. These numbers represent the factors in the model. All of the variables represent a potential factor, so there are as many factors as variables in the left

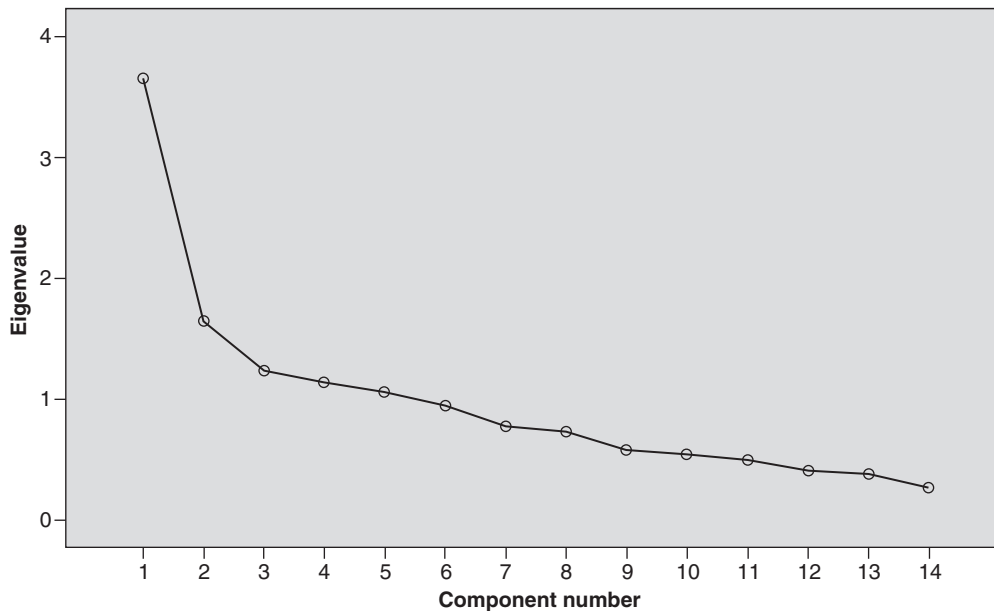**TABLE 14-4**   **Total Variance Explained**

Total Variance Explained

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings		
	Total	**% of Variance**	**Cumulative %**	**Total**	**% of Variance**	**Cumulative %**
1	3.661	26.151	26.151	3.661	26.151	26.151
2	1.655	11.823	37.973	1.655	11.823	37.973
3	1.246	8.903	46.877	1.246	8.903	46.877
4	1.158	8.271	55.148	1.158	8.271	55.148
5	1.072	7.658	62.806	1072	7.658	62.806
6	.961	6.868	69.674			
7	.786	5.616	75.289			
8	.744	5.316	80.606			
9	.588	4.198	84.804			
10	.552	3.944	88.748			
11	.504	3.603	92.351			
12	.414	2.960	95.311			
13	.384	2.742	98.053			
14	.273	1.947	100.000			

Extraction Method: Princloal Comoonent Analysis.

columns of the table. In the right 3 columns of the table, only the factors that contribute to the model are included (factors 1-5).

The factors in the principal component analysis show individual relationships, much like the beta values in regression. In fact, the factor loadings here are the correlations between the factors and their related variables. The **Eigenvalue** used to establish a cutoff of factors is a value like R^2 in regression. As with regression, the Eigenvalue represents the "strength" of a factor. The Eigenvalue of the first factor is such that the sum of the squared factor loadings is the most for the model. The reason the Eigenvalue is used as a cutoff is because it is the sum of the squared factor loadings of all variables (the sum divided by the number of variables in a factor equals the average percentage of variance explained by that factor). Since the squared factor loadings are divided by the number of variables, an Eigenvalue of 1 simply means that the variables explain at least an average amount of the variance. A factor with an Eigenvalue of less than 1 means the variable is not even contributing an average amount to explaining the variance.

It is also common to evaluate the **scree plot** to determine how many factors to include in a model. The scree plot is a graphical representation of the incremental variance accounted for by each factor in the model. An example of a scree plot is shown in **Figure 14-1**. To use the scree plot to determine the number of factors in the model, look at where the scree plot begins to level off. Any factors that are in the level part of

Figure 14-1 Scree Plot

the scree plot may need to be excluded from the model. Excluding variables this way should be thoroughly supported both theoretically and methodologically.

Figure 14- 1 is a good example of where a scree plot may be in conflict with the eigenvalue cutoff, and where it may be appropriate to include factors that have an Eigenvalue of less than 1. Here, the plot seems to level off about the eighth factor, even though the Eigenvalue cutoff is reached after the fifth factor. This is somewhat supported by **Table 14-5**, which shows the initial Eigenvalues. This shows that the Eigenvalues keep dropping until the eighth factor and then essentially level off to Eigenvalues in the 0.2 to 0.5 range. This shows where the leveling occurs in the model. In this case, it may be beneficial to compare a factor model based on an Eigenvalue of 1 cutoff to a scree plot cutoff to see which model is more theoretically supportable.

It is also important in the extraction phase to examine the communality. The communality is represented by the sum of the squared loadings for a variable across factors. The communalities can range from 0 to 1. A communality of 1 means that all of the variance in the model is explained by the factors (variables). This is shown in the "Initial" column of **Table 14-6**. The initial values are where all variables are included in the model. They have a communality of 1 because there are as many variables as there are factors. In the "Extraction" column, the communalities are different and less than 1. This is because only the 5 factors used above (with Eigenvalues greater than 1) are taken into account. Here the communality for each variable as it relates to one of the five factors is taken into account. Although there are no 0 values; if there were, it would mean that variable (factor) contributed nothing to explaining the common variance of the model.

At this point and after rotation (see below), you should examine the **factor matrix** (**component matrix**) to determine what variables could be combined (those that load together) and if any variables should be dropped. This is accomplished through the

TABLE 14-5   Communalities

	Initial	Extraction
Recoded ARREST varible to represent yes or no	1.000	686
Do you ever go to church?	1.000	.693
Are you involved in any clubs or sports in school?	1 000	600
Do you have acurfew at home?	1.000	.583
Scale measure of gang activities	1.000	.627
Have you ever been grounded because of drugs or alcohol?	1.000	.405
Do you regularly carry a gun with you?	1.000	694
Have your parents ever threatened to throw you out of the house because of drugs or alcohol?	1.000	.728
Have you ever carried a gun out with you when you went out at night?	1.000	.636
Have you ever been in a group where someone was carrying a gun?	1.000	.646
Recoded UCONVIC variable to represent yes or no	1.000	.774
Have any of your arrests involved a firearm?	1.000	.785
Recoded SKIPPED variable to yes and no	1.000	.410
Have you ever been in trouble at school because of drugs or alcohol?	1.000	.528

Extraction Method: PrinciDal Comoonent Analysis.

TABLE 14-6   Component Matrix

	Initial	Extraction
ARRESTR	1.000	.686
CHURCH	1.000	.693
CLUBS	1.000	.600
CURFEW	1.000	.583
GANGR	1.000	.627
GRND_DRG	1.000	.405
GUN_REG	1.000	.694
OUT_DRUG	1.000	.728
OUT_GUN	1.000	.636
GRUP_GUN	1.000	.646
UCONVICR	1.000	.774
U_GUN_CM	1.000	.785
SKIPPEDR	1.000	.410
SCH_DRUG	1.000	.528

Extraction Method: Principal Component Analysis.

TABLE 14-7   **Component Matrix with Blocked Out Values**

	Component				
	1	**2**	**3**	**4**	**5**
GANGR	.751				
GUN_REG	.643	.424			
U_GUN_CM	.637				−.429
UCONVICR	.634				−.405
OUT_GUN	.627				
SCH_DRUG	.494		.442		
SKIPPEDR	−.452	.413			
CHURCH		.525		.514	
GRND_DRG		−.403			
OUT_DRUG	.520		.534		
ARRESTR		−.473		.631	
CURFEW			.433	.463	
CLUBS					.527
GRUP_GUN	.467				.523

Extraction Method: Principal Component Analysis.

a 5 components extracted.

Factor Loading Value. This is the correlation between a variable and a factor where only a single factor is involved or multiple factors are orthogonal (in regression terms, it is the standardized regression coefficient between the observed values and common factors). Higher factor loadings indicate that a variable is closely associated with the factor. Look for scores greater than 0.40 in the factor matrix. In fact, most statistical programs allow you to block out factor loadings that are less than a particular value (less than 0.40). This is not required, but it makes the factor matrix more readable, and was done in **Table 14-7**.

Step 4—Factor Rotation

It is possible, and acceptable, to stop at this point and base analyses on the extracted factors. Typically, however, the extraction in the previous step is subjected to an additional procedure to facilitate a clearer understanding of the data. As shown in Figure 14-7, the variables are related to factors seemingly at random. It is difficult to determine clearly which variables load together. Making this interpretation easier can be accomplished by rotating the factors. That is the next step in the factor analysis process. *Factor rotation* simply rotates the ordinate plane so the geometric location of the factors makes more sense (see **Figure 14-2**). As shown in this figure, some of the factors (the dots) are in the positive, positive portion of the ordinate plane and some are in the positive, negative portion. This makes the analysis somewhat difficult. By rotating the plane, however, all of the factors can be placed in the same quadrant. This
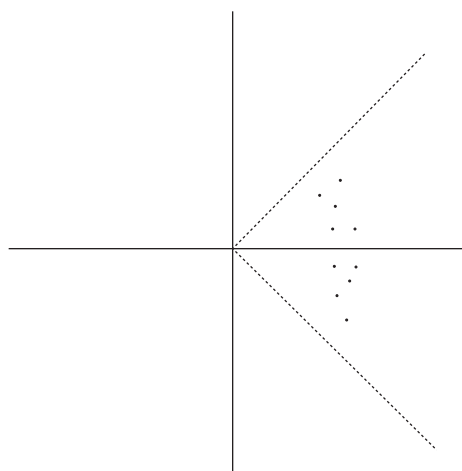
Figure 14-2 Factor Plot in Coordinate Planes

makes the interpretation much simpler, but the factors themselves have not been altered at all.

This step involves, once again, examining the factor matrix. Values should be at least .40 to be included in a particular factor. For those variables that reach this cutoff, it is now possible to determine which variables load (group) with other variables. If this is an exploratory factor analysis, this is where a determination can be made concerning which variables to combine into scales or factors. If this is a confirmatory factor analysis, this step will determine how the theoretical model faired under testing.

There are two main categories of rotations: *orthogonal* and *oblique*. There are also several types of rotations available within these categories. It should be noted that, although rotation does not change the communalities or percentage of variation explained, each of the different types of rotation strategies may produce different mixes of variables within each factor.

The first type of rotation is an orthogonal rotation. There are several orthogonal rotations that are available. Each performs a slightly different function, and each has its advantages and disadvantages. Probably the most popular orthogonal procedure is **varimax** (developed by Kaiser in his Ph.D. dissertation and published in Kaiser 1958). This rotation procedure attempts to minimize the number of **variables** that have high loadings on a factor (thus achieving the goal of parsimony discussed above). Another rotation procedure, **quartermax**, attempts to minimize the number of **factors** in the analysis. This often results in an easily interpretable set of variables, but where there are a large number of variables with moderate factor loadings on a single factor. A combination or compromise of these two is found in the **equamax** rotation, which attempts to simplify both the number of factors and the number of variables.

An example of a varimax rotation is shown in **Table 14-8**. As with the principal component analysis, the values less than 0.40 have been blanked out. Here, the five factors identified in the extraction phase have been retained, along with the variables from the component matrix in Table 14-7. The difference here is that the structure of the model is clearer and more interpretable.

This table shows that the variables OUT_DRUG (respondent reported being out with a group where someone was using drugs), GUN_REG (respondent carries a gun
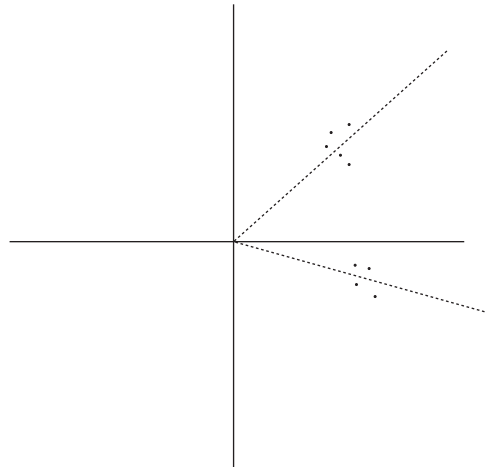
TABLE 14-8   **Rotated Component Matrix**

	Component				
	1	**2**	**3**	**4**	**5**
OUT_DRUG	.848				
GUN_REG	.778				
GANGR	.503				
GRUP_GUN		.776			
OUT_GUN		.687			
UCONVICR			.789		
U_GUN_CM			.741		
CURFEW				.447	
CHURCH				.819	
SKIPPEDR				.539	
CLUBS				.539	
ARRESTR					.783
SCH_DRUG					.524
GRND_DRG					.524

Extraction Method: Principal Component Analysis. Rotation Method: Varimax with Kaiser Normalization.

regularly) and GANGR (respondent reported being in a gang) are the variables that load together to represent Factor 1; GRUP_GUN (respondent reported being out with a group where someone was carrying a gun) and OUT_GUN (respondent has previously carried a gun out with them) load together to represent Factor 2; UCONVICR (respondent had been convicted of a crime) and U_GUN_CM (respondent had been convicted of a crime involving a gun) load together to represent Factor 3; CURFEW (respondent has a curfew at home) CHURCH (respondent regularly attends church), SKIPPEDR (respondent had skipped school because of drugs) and CLUBS (respondent belongs to clubs or sports at school) load together to represent Factor 4; and ARRESTR (respondent has been arrested before), SCH_DRUG (respondent has been in trouble at school because of drugs)and GRND_DRG (respondent has been grounded because of drugs) load together to represent Factor 5.

A second type of rotation is an oblique rotation. The oblique rotation used in SPSS is oblimin. This rotation procedure was developed by Carroll in his 1953 work and finalized in 1960 in a computer program written for IBM mainframe computers. Oblique rotation does not require the axes of the plane to remain at right angles. For an oblique rotation, the axes may be at almost any angle that sufficiently describes the model (see **Figure 14-3**).

Figure 14-3 Graph of Oblique Rotation

Early in the development of factor analysis, oblique rotation was considered unsound as it was a common perception that the factors should be uncorrelated with each other. Thurstone began to change this perception in his 1947 work, in which he argued that it is unlikely that factors as complicated as human behavior and in a world of interrelationships such as our society could truly be unrelated such that orthogonal rotations alone are required. It has since become more accepted to use oblique rotations under some circumstances.

The way an oblique rotation treats the data is somewhat similar to creating a least-squares line for regression. In this case, however, two lines will be drawn, representing the ordinate axes discussed in the orthogonal rotation. The axes will be drawn so that they create a least-squares line through groups of factors (as shown in Figure 14-3).

As shown in this graph, the axis lines are not at right angles as they are with an orthogonal rotation. Additionally, the lines are oriented such that they run through the groups of factors. This illustrates a situation where an oblique rotation is at maximum effectiveness. The closer the factors are clustered, especially if there are only two clusters of factors, the better an oblique rotation will identify the model. If the factors are relatively spread out, or if there are three or more clusters of factors, an oblique rotation may not be as strong a method of interpretation as an orthogonal rotation. That is why it is often useful to examine the factor plots, especially if planning to use an oblique rotation.

Although there are similarities between oblique and orthogonal rotations (i.e. they both maintain the communalities and variance explained for the factors), there are some distinct differences. One of the greatest differences is that, with an oblique rotation, the factor loadings are no longer the same as a correlation between the variable and the factor because the factors are not independent of each other. This means that the variable/factor loadings may span more than one factor. This makes interpretation of what variables load on which factors more difficult because the factoral complexity is materially increased. As a result, most statistical programs, including SPSS, incorporate both a factor loading matrix (pattern matrix) and a factor structure matrix for an oblique rotation.

The interpretation of an oblique rotation is also different than with an orthogonal rotation. It should be stressed, however, that all of the procedures up to the rotation phase are the same for both oblique and orthogonal rotations. It should also be noted that an orthogonal rotation can be obtained using an oblique approach. If the best angle of the axes happened to be at 90 degrees, the solution would be orthogonal even though an oblique rotation was used.

The difference in the output between an orthogonal and an oblique rotation is that the pattern matrix for an oblique rotation contains negative numbers. This is not the same as a negative correlation (implying direction). The angle in which the axes are oriented is measured by a value of delta (f) in SPSS. The value of f is at 0 when the factors are most oblique; and negative value of f means the factors are less oblique. Generally, negative values of are preferred, and the more negative the better. This means that, for positive values, the factors are highly correlated; for negative values, the factors are less correlated (less oblique); and when the values are negative and large, the factors are essentially uncorrelated (orthogonal).

Table 14-9 does not contain a lot of negative values. There are also a fairly large number of values much greater than zero. These variables should be carefully examined to determine if they should be deleted from the model; and the value of using an oblique rotation in this case may need to be reconsidered.

TABLE 14-9   **Pattern Matrix**

	Component				
	1	2	3	4	5
GANGR	.418	.048	.433	.144	−.253
GUN_REG	.092	−.026	.760	−.141	−.164
U_GUN_CM	.127	.210	.313	−.164	−.730
UCONVICR	.101	−.031	−.078	.276	−.792
OUT_GUN	.666	.143	.244	−.038	−.146
SCH_DRUG	−.023	−.087	.480	.483	.008
SKIPPEDR	−.152	.510	−.050	−.174	.101
CHURCH	−.003	.836	−.090	.018	−.111
GRND_DRG	.050	−.304	.123	.487	.075
OUT_DRUG	−.046	−.070	.869	−.040	.077
ARRESTR	.069	.084	−.229	.801	−.168
CURFEW	−.516	.421	.152	.311	.149
CLUBS	.281	.492	.016	−.005	.550
GRUP_GUN	.800	−.080	−.059	.129	.112

Extraction Method: Principal Component Analysis.

Rotation Method: Oblimin with Kaiser Normalization.

a Rotation converged in 20 iterations.

TABLE 14-1O   Structure Matrix

	Component				
	1	2	3	4	5
GANGR	.568	−.088	.576	.229	−.441
GUN_REG	.274	−.089	.798	−.041	−.329
U_GUN_CM	.326	.088	.451	−.099	−.777
UCONVICR	.296	−.199	.128	.340	−.826
OUT_GUN	.732	.030	.395	.061	−.330
SCH_DRUG	.114	−.185	.528	.542	−.131
SKIPPEDR	−.256	.572	−.151	−.273	.238
CHURCH	−.089	.822	−.121	−.107	.034
GRND_DRG	.126	−.378	.187	.542	−.045
OUT_DRUG	.112	−.103	.845	.050	−.091
ARRESTR	.110	−.053	−.107	.783	−.187
CURFEW	−.547	.447	.026	.216	.282
CLUBS	.098	.543	−.069	−.098	.556
GRUP_GUN	.780	−.170	.095	.183	−.087

Extraction Method: Principal Component Analysis.
Rotation Method: Oblimin with Kaiser Normalization.

The structure matrix for an oblique rotation displays the correlations between factors. An orthogonal rotation does not have a structure matrix because the factors are assumed to have a correlation of 0 between them (actually the pattern matrix and the structure matrix are the same for an orthogonal rotation). With an oblique rotation, it is possible for factors to be correlated with each other and variables to be correlated with more than one factor. This increases the factoral complexity, as discussed above, but it does allow for more complex relationships that are certainly a part of the intricate world of human behavior.

As **Table 14-10** shows, GANGR loads fairly high on Factors 1 and 3. This is somewhat supported by the direct relation between GANGR and these factors shown in the pattern matrix; but the values in the structure matrix are higher. This is a result of GANGR being related to both factors; thus part of the high value between GANGR and Factor 3 is being channeled through Factor 1 (which essentially serves as an intervening variable).

Step 5—Use of Factors in Other Analyses

After completing a factor analysis, the factors can be used in other analyses, such as including the factors in a multiple regression. The way to do this is to save the factors as variables; therefore the factor values become the values for that variable. This creates a kind of scale measure of the underlying dimension that can be used as a measure of the concept in further analyses. Be careful of using factors in regression, however. Not that

it is wrong, but factored variables often have a higher R^2 than might be expected in the model. This is because each of the factors is a scale measure of the underlying dimension. A factor with three variables known to be highly correlated will naturally have a higher R^2 than separate variables that may or may not be correlated.

Another problem with factors being utilized in multiple regression analysis, is the interpretation of a factor that contains two variables, if not more. In this case, the b-coefficient is rendered useless. Because of the mathematical symbiosis of the variables, the only coefficient that can adequately interpret the relationship is the standardized coefficient (beta).

Factor analysis is often a great deal of work and analysis. Because of this, and the advancement of other statistical techniques, factor analysis has fallen largely into disuse. While originally factor analysis' main competition was path analysis, now structural equation modeling (SEM) has taken advantage on both statistical approaches.

■ Structural Equation Modeling

Structural equation modeling (SEM) is a multi-equation technique in which there can be multiple dependent variables. Recall that in all forms of multiple regression, we used a single formula ($y = a + bx + e$) with one dependent variable. Multiple equation systems allow for multiple indicators for concepts. This requires the use of matrix algebra, however, which greatly increases the complexity of the calculations and analysis. Fortunately, there are statistical program such as AMOS or LISREL that will perform the calculations, so we will not address those here. Byrne (2001) explores SEM using AMOS and Hayduk (1987) examines SEM through usingLISREL. While the full SEM analysis is beyond the scope of this book, We will address how to set up a SEM model and understand some of the key elements.

Structural equation models consist of two primary models: Measurement (null) and structural models. The measurement model pertains to how observed variables relate to unobserved variables. This is important in the social sciences as every study is going to be missing information and variables. As discussed in Chapter 2, there are often confounding variables in research that are not included in the model or accounted for in the analysis, but which have an influence on the outcome (generally through variables that are included in the model). Structural Equation Models deal with how concepts relate to one another and attempts to account for these confounding variables. An example of this is the relationship between socioeconomic status (SES) and happiness. In theory, the more SES you have, the happier you should be; however, it is not the SES that make you happy but the confounding variables like luxury items, satisfactory job, etc. that may be making this relationship.

SEM addresses this kind of complex relationship by creating a theoretical model of the relationship that is then tested to see if the theory matches the data. As such, SEM is a confirmatory procedure; a model is proposed, a theoretical diagram is generated, and an examination of how close the data is to the model is completed. The first step in creating a SEM is putting the theoretical model into a path diagram. It may be beneficial to review the discussion of path models presented in Chapter 3 as these will be integral to the discussion of SEM that follows. In SEM, we create a path diagram based on theory and then place the data into an SEM analysis to see how close the analysis is to what was expected in the theoretical model. We want the two models to not be statistically significantly different.

Variables in Structural Equation Modeling

Like many of the analysis procedures discussed throughout this book, SEM has a specific set of terms that must be learned. Most importantly in SEM, we no longer use the terms dependent and independent variables since there can be more than one dependent variable in an analysis. For the purposes of SEM, there are two forms of variables: Exogenous and endogenous. Exogenous variables are always analogous to independent variables. Endogenous variables, on the other hand, are variables that are at some point in the model a dependent variable; while at other points they may independent variables.

SEM Assumptions

Five assumptions must be met for structural equation modeling to be appropriate. Most of these are similar to the assumptions of other multivariate analyses. First, the relationship between the coefficients and the error term must be linear. Second, the residuals must have a mean of zero, be independent, be normally distributed, and have variances that are uniform across the variable. Third, variables in SEM should be continuous, interval level data. This means SEM is often not appropriate for censored data. The fourth assumption of SEM is no specification error. As noted above, if necessary variables are omitted or unnecessary variables are included in the model, there will be measurement error and the measurement model will not be accurate. Finally, variables included in the model must have acceptable levels of kurtosis. This final assumption bears remembering. An examination of the univariate statistics of variables will be key when completing a SEM. Any variables with kurtosis values outside the acceptable range will produce inaccurate calculations. This often limits the kinds of data often used in criminal justice and criminology research. While scaled ordinal data is sometimes still used in SEM (continuity can be faked by adding values), dichotomous variables often have to be excluded from SEM as binary variables have a tendency to suffer from unacceptable kurtosis and non-normality.

Advantages of SEM

There are three primary advantages to SEM. These relate to the ability of SEM to address both direct and indirect effects, the ability to include multi-variable concepts in the analysis, and inclusion of measurement error in the analysis. These are addressed below.

First, SEM allows researcher to identify direct and indirect effects. Direct effects are principally what we look for in research—the relationship between a dependent variable we are interest in explaining (often crime/delinquency) and an independent variable we think is causing or related to the dependent variable. This links directly from a cause to an effect. For example:

ART TO COME

The hallmark of a direct effect is that an arrow goes from one variable only into another variable. In this example, delinquent peers are hypothesized to have a direct influence on an individual engaging in crime.

An indirect effect occurs when one variable goes through another variable on the way to some dependent or independent variable. For example:

ART TO COME

A path diagram of indirect effects indicates that age has an indirect influence on crime by contributing the type of peers one would associate with on a regular basis.
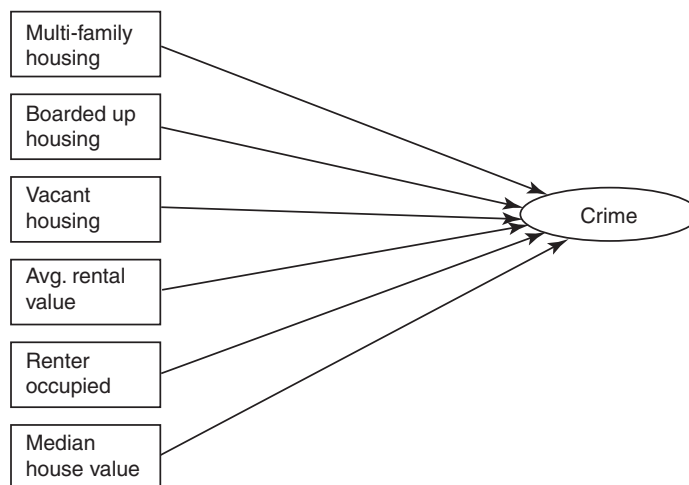
Complicating SEM models, it is also possible for variables in structural equation modeling to have both direct and indirect effects at the same time. For example:
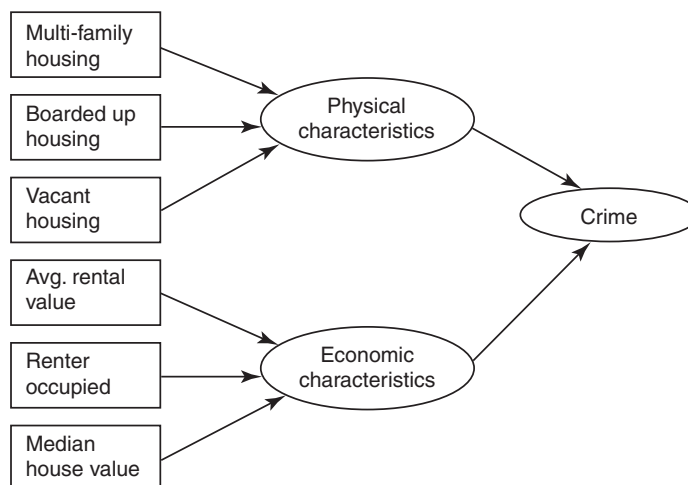
ART TO COME

In this case, age is hypothesized to have a direct effect on crime (age-crime curve), but it also has an indirect effect on crime through the number of delinquent peers with which a juvenile may keep company. These are the types of effects that are key in SEM analysis.

The second advantage of SEM is that it is possible to have multiple indicators of a concept. The path model for a multivariate analysis based on regression might look like **Figure 14-4.**

With multiple regression analysis we only evaluate the effect of individual independent variables on the dependent variable. While this creates a fairly simple path

Figure 14-4 Path Model of Regression Analysis on the Effect of Social Disorganization on Crime at the Block Level

Figure 14-5 Path Model Oof SEM Analysis on the Effect of Social Disorganization on Crime at the Block Level

model, it is not necessarily the way human behavior works. SEM allow for the estimation of the combined effects of independent variables into concepts/constructs. **Figure 14-5** illustrates the same model as Figure 14-4 but with the addition of theoretical constructs linking the independent variable to the dependent variable.

As shown in Figure 14-5, the variables are no longer acting alone, but in concert with like variables that are expected conceptually to add to the prediction of the dependent variable. In the case of social disorganization, physical and economic characteristics of a neighborhood are predicted to contribute to high levels of crime.

The final advantage of SEM is that it includes measurement error into the model. As discussed above, path analyses often use regression equations to test a theoretical causal model. A problem for path analyses using regression as its underlying analysis method is that, while these path analyses include error terms for prediction, they do not adequately control for measurement error. SEM analyses do account for measurement error, therefore providing a better understanding of how good the theoretical model predicts actual behavior. For example, unlike the path analysis in Figure 14-4, **Figure 14-6** indicates the measurement error in the model.
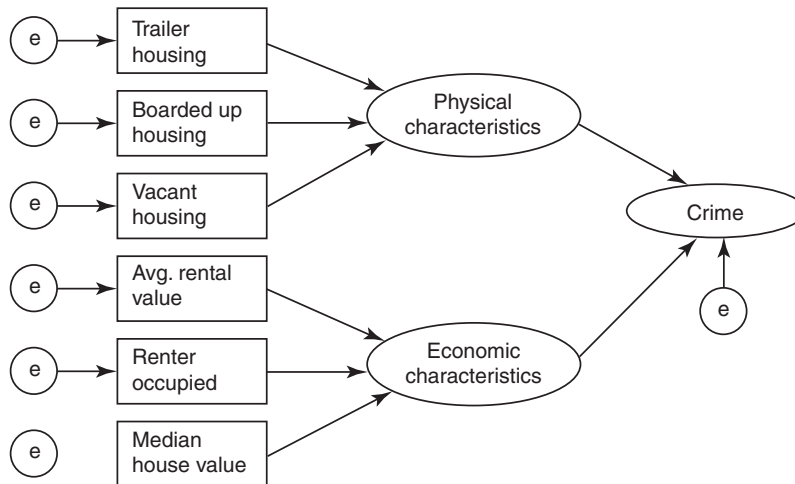
The e represents the error associated with the measurement error of each measured variable in the structural equation model. Notice there are no error terms associated with the constructs of physical and economic characteristics. This is because these concepts are a combination of measured variables and not measures themselves.

SEM Analysis

There are six steps in conducting a structural equation model analysis. These are listed in **Table 14-11.**

The first part of any SEM is specifying the theoretical/statistical model. This part of the process should begin even prior to collecting data. Proper conceptualization, operationalization, and sampling are all key parts of Step 1.

Once conceptualization and model building are complete, the second step is to develop measures expected to be representative of the theoretical model and to collect

Figure 14-6 The Inclusion of Error Terms

TABLE 14-11   **Steps in SEM Analysis Process**

Step 1	Specify the model
Step 2	Select measures of the theoretical model and collect the data
Step 3	Determine whether the model is identified
Step 4	Analyze the model (something missing here)
Step 5	Evaluate the model fit (How well does the model account for your data?)
Step 6	If the original model does not work, respecify the model and start again

data. This is more of an issue for research methods than statistics, so you should consult a methodology book for guidance here.

The third step in structural equation modeling is determining if the model is identified. **Identification** is a key idea in SEM. Identification deals with issues of whether there is enough information (variables) and if that information is distributed across the equations in such a way to estimate coefficients and matrices that are not known (Bollen, 1989). If a model is not identified in the beginning of the analysis, the theoretical must be changed or the analysis abandoned. There are three types of identified models: Overidentified, just identified, and underidentified. Overidentified models permit tests of theory. This is the desired type of model identification. Just identified models are considered not interesting. These are the types of models researchers deal with when using multiple regression analyses. Underidentified models suggest that we can not do anything with the model until it has been re-specified or additional information has been gathered.

There are several ways to test identification. First, is the t-Rule. This test provides necessary, but not sufficient conditions for identification. If test is not passed, the model is not identified. If the test is passed, the model still may not be identified. The Null B Rule applies to models in which there is no relationship between the endogenous variables, which is rare. The Null B Rule provides sufficient, but not necessary

conditions for identification. This test is not used often, with the exception of no rela-tionships existing between endogenous variables. The Recursive Rule is sufficient but not necessary for identification. To be recursive, there must not be any feedback loops among endogenous variables. An OLS regression is an example of a non-recursive model as long as there are no interaction terms involved. The Recursiv rule does not help establish identification for models with correlated errors. The Order Condition Test suggests that the number of variables excluded from each equation must be at least P-1, where P is the number of equations. If equations are related, then the model is underidentified. This is a necessary, but not a sufficient condition to claim identifica-tion. The final test of identification is the Rank Condition test. This test is both neces-sary and sufficient, making it one of the better tests of identification. Passing this test means the model is appropriate for further analyses.

The fourth step in SEM analysis is analyzing the model. Unlike multiple regression analyses, SEM utilizes more than one coefficient for each variable. SEM is therefore based on matrix algebra with multiple equations. The primary matrices used in SEM are the covariance matrix and variance-covariance matrix. These are divided into four matrices of coefficients and four matrices of covariance. The four matrices of coeffi-cients are: 1) a matrix that relates the endogenous concepts to each other (β); 2) a matrix that relates exogenous concepts to endogenous concepts (γ); 3) a matrix that relates endogenous concepts to endogenous indicators (λy); and, 4) a matrix that relates exogenous concepts to exogenous indicators (λx). Once these relationships have been examined for the endogenous and exogenous variables, the covariance among the variables must be examined. This is accomplished through the four matrices of covari-ance, which are: 1) covariance among exogenous concepts (Φ); 2) covariance among errors for endogenous concepts (Ψ); 3) covariance among error for exogenous indica-tors ($\Theta\delta$) and, 4) covariance among error for endogenous indicators ($\Theta\epsilon$). Within these models, we are interested in three statistics, the mean, variance, and covariance.

While much of SEM analysis output is similar to regression output (R^2, b-coeffi-cients, standard errors, and standardized coefficients), it is much more complicated. In SEM, researchers have to contend with, potentially, four different types of coefficients and four different types of covariances. This typically means having to examine up to twenty pages of output. As discussed below, due to space limitations and the complex-ity of the interpretation, a full discussion of SEM output is not discussed in this book. Anyone seeking to understand how to conduct an SEM should take a class strictly on this method. As was indicated above, however, the most important element of SEM is its ability to evaluate an overall theoretical model. This portion of the anlaysis is briefly discussed below.

The key equation in SEM is the basis for the structural model. This equation is used in each of the eight matrices. The formula to examine the structural model is:

$$\eta = \beta\eta + \gamma\xi + \zeta$$

Where η is the endogenous (latent) variable(s), ξ is the exogenous (latent) variable(s), β are coefficients of endogenous variables, γ are coefficients of exogenous variables, and ζ is any error among endogenous variables. This is the model that underlies struc-tural equation model analysis.

The other two equations that need to be calculated are the exogenous measurement model and the endogenous measurement model. The formula for the exogenous measurement model is:

$$X = \lambda_X \xi + \delta$$

Where X is an exogenous indicator, λx is thecoefficient from exogenous variables to exogenous indicators, ξ is the exogenous latent variable(s), and δ is error associated with the exogenous indicators. The endogenous measurement model is:

$$Y = \lambda_y \eta + \epsilon$$

Where Y is an endogenous indicator, λy is the coefficient from endogenous variables to endogenous indicators, η is the endogenous latent variable(s), and ϵ is error associated with the endogenous indicators.

These matrices and equations will generate the coefficients associated with the structural equation model analysis. After all of these have been calculated, it is necessary to examine which model can better predict the endogenous variables, the structural model, or the measurement model. This is accomplished by using either Chi-Square or an Index of Fit. Remember that when using Chi-Square as the method to determine the validity of the model, you want a value associated less than 0.05. If the Chi-Square is significant, it means that the hypothesized model is better than if it had been "just identified."

Besides Chi-Square, there are ten different **Indices of Fit** to choose from to determine how well the theoretical model is at predicting endogenous variables. **Table 14-12** provides the list of Indices of Fit used in SEM.

A full discussion of these is beyond this book. Suffice it to say that, GFI and the AGFI produces a score between 0 and 1. A score of 0 implies poor model fit; a score of 1 implies perfect model fit. These measures are comparable to R^2 values in regression. The

TABLE 14-12   SEM Indices of Fit

Shorthand	Index of Fit
GFI	Goodness of Fit
AGFI	Adjusted Goodness of Fit
RMR	Root Mean Square Residual
RMSEA	Root Mean Square Error of Approximation
NFI	Normal Fit Index
NNFI	Non-Normal Fit
IFI	Incremental Fit
CFI	Comparative Fit
ECVI	Expected Cross Validation
RFI	Relative Fit

NFI, NNFI, IFI, and the CFI all indicate the proportion in improvement of the proposed model relative to the null model. A high value here indicates better fit of the model.

The sixth step in the SEM analysis is a decision of the contribution of the model. If the model has been shown to be adequate, the theoretical model is supported and you have "success." If the model was not adequate (underspecified), you must either abandon the research or go back and re-conceptualize the model and begin again.

There is not enough room in this chapter to provide a full example of SEM output. Even in a simplistic example, the amount of output is almost twenty pages in length. This discussion is meant to give you a working knowledge of structural equation modeling such that you can understand its foundation when reading journal articles or other publications.

■ Conclusion

This chapter introduced you to multivariate techniques beyond multiple regression analysis, principally factor analysis and structural equation modeling. This short review does not represent a full understanding of these multivariate statistical analyses. These procedures are complicated and require a great deal of time and effort to master. You should expect to take a course on each of these and complete several research projects of your own before you begin to truly understand them.

This chapter brings to an end the discussion of multivariate statistical techniques. Although other analytic strategies are available, such as hierarchal linear modeling (HLM) and various time-series analyses, this book is not the proper platform for a description of these techniques. In the next chapters of the book, we address how to analyze data when you do not have a population and must instead work with a sample. This is inferential statistical analysis.

■ Key Terms

Anti-image correlation matrix	Identification
Bartlett's test of sphericity	Identity matrix
Beta	Indices of Fit
Communality	Matrix
Component matrix	Matrix algebra
Confirmatory factor analysis	Oblimin
Covariance	Oblique
Eigenvalue	Orthogonal
Endogenous variables	Path analysis
Error variance	Pattern matrix
Exogenous variables	Principal components analysis
Exploratory factor analysis	Scree plot
Factor analysis	Structural equation modeling
Factor loading value	Structure matrix
Factor matrix	Uniqueness
Factor rotation	Varimax
Factorial complexity	

Key Equations

Slope-Intercept/Multiple Regression Equation

$$y = a + bx + e$$

The Structural Model

$$\eta = \beta\eta + \gamma\xi + \zeta$$

The Exogenous Measurement Model

$$X = \lambda_X \xi + \delta$$

The Endogenous Measurement Model

$$Y = \lambda y \eta + \epsilon$$

Questions and Exercises

1. Select journal articles in criminal justice and criminology that have as their primary analysis factor analysis, path analysis, or structural equation modeling (it will probably require three separate articles). For each of the articles:
   a. discuss the type of procedure employed (for example, an oblique factor analysis).
   b. trace the development of the concepts and variables to determine if they are appropriate for the analysis (why or why not?).
   c. before looking at the results of the analysis, attempt to analyze the output and see if you can come to the same conclusion(s) as the article.
   d. discuss the limitations of the analysis and how it differed from that outlined in this chapter.
2. Design a research project as you did in Chapter 1 and discuss which of the statistical procedures in this chapter would be best to use to analyze the data. Discuss what limitations or problems you would expect to encounter.

References

Bollen, K.A. 1989. *Structural Equations with Latent Variables.* New York: John Wiley and Sons.

Byrne, B.M. 2001. *Structural Equation Modeling With AMOS: Basic Concepts, Applications and Programming.* New Jersey: Lawrence Erlbaum Associates, Publishers.

Carroll, J. B. 1953. Approximating Simple Structure in Factor Analysis. *Psychometrika,* 18:23–38.

Duncan, O.D. 1966. Path analysis: Sociological Examples. *American Journal of Sociology* 73:1–16.

Garnett, J. C. M. 1919. On Certain Independent Factors in Mental Measurement. *Proceedings of the Royal Society of London,* 96:91–111.

Hatcher, L. 1994. *A Step-by-Step Approach to Using the SAS System for Factor Analysis and Structural Equation Modeling.* Cary, NC:SAS Institute, Inc.

Hayduk, L.A. 1987. *Structural Equation Modeling with LISREL: Essentials and Advances.* Baltimore: The Johns Hopkins University Press.

Hotelling, H. 1933. Analysis of a Complex of Statistical Variables into Principal Components. *Journal of Educational Psychology*, 24:417&.

Kaiser, H. F. 1958. The Varimax Criterion for Analytic Rotation in Factor Analysis. *Psychometrika*, 23:187–200.

Kaiser, H. F. 1974a. An Index of Factorial Simplicity. *Psychometrika*, 39:31–36.

Kaiser, H. F. 1974b. A Note on the Equamax Criterion. *Multivariate Behavioral Research*, 9:501–503.

Kim, J.O., and C.W. Mueller. (1978a). *Introduction to Factor Analysis.* London: Sage Publications.

Kim, J.O., and C.W. Mueller. (1978b). *Factor Analysis: Statistical Methods and Practical Issues.* London: Sage Publications.

Kline, R.B. 1998. *Principles and Practice of Structural Equation Modeling.* New York: The Guilford Press.

Lawley, D. N. 1940. The Estimation of Factor Loadings by the Method of Maximum Likelihood. *Proceedings of the Royal Society of Edinburgh*, 60:64–82.

Long, L.S. 1983. *Confirmatory Factor Analysis.* London: Sage Publications.

Pearson, K. 1901. On Lines and Planes of Closest Fit to System of Points in Space. *Philosophical Magazine*, 6(2):559–572.

Spearman, C. 1904. General Intelligence, Objectively Determined and Measured. *American Journal of Psychology*, 15:201–293.

Wright, S. 1934. The Method of Path Coefficients. *Annals of Mathematical Statistics*, 5: 161–215.

Thurstone, L. L. 1931. Multiple Factor Analysis. *Psychological Review*, 38:406–427.

Thurstone, L. L. 1947. *Multiple Factor Analysis.* Chicago: University of Chicago Press.